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AA—97—2022

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2022

(CBCS/New Pattern)

MATHEMATICS

Paper XIII

(Linear Algebra)

(Friday, 16-12-2022) Time : 10.00 a.m. to 12.00 noon

Time—Two Hours *Maximum Marks—40*

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Illustrate your answers with suitably labelled diagram, wherever necessary.

1. If U and W are two subspaces of a finite-dimensional vector space V, then prove that : 15

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W).$$

Or

Answer each of the following : 8

(*a*) In V_2, show that (3, 7) belongs to [(1, 2), (0, 1)] but does not belong to [(1, 2), (2, 4)].

(*b*) For a vector space V prove each of the following : 7

(*i*) $\alpha 0 = 0$ for every scalar α

(*ii*) $0u = u$ for every $u \in V$

(*iii*) $(-1)u = -u$ for every $u \in V$.

P.T.O.

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2. If $T : U \rightarrow V$ and $S : V \rightarrow W$ are two linear maps, then prove each of the following : 15

(i) If S and T are non-singular, then ST is non-singular and $(ST)^{-1} = T^{-1}S^{-1}$.

(ii) If ST is one-one, then T is one-one.

(iii) If ST is onto, then 'S' is onto.

Or

Answer each of the following :

(a) Prove that any orthogonal set of non-zero vectors in an inner product space is linearly independent. 8

(b) Find all the eigenvalues of : 7

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}.$$

3. Attempt any *two* of the following : 10

(a) Prove that the vectors (1, 1, 1), (1, 1, 0) and (1, 0, 1) in V_3 are linearly independent.

(b) Define a basis for a vector space and prove that in an *n*-dimensional vector space 'V' any set of '*n*' linearly independent vector is a basis.

(c) Prove that if $T : U \rightarrow V$ and $S : V \rightarrow W$ are two linear maps, then their composition is also linear map.

(d) If V is an inner product space, then prove that for any vectors '*u*' and *v* in V :

$$|u.v| \leq \|u\| . \|v\|.$$