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GA—121—2023

FACULTY OF SCIENCE

B.Sc. (Fourth Semester) EXAMINATION

APRIL/MAY, 2023

(New Course)

MATHEMATICS

Paper XI

(Partial Differential Equations)

(Thursday, 11-5-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory

(*ii*) Figures to the right indicate full marks.

1. Discuss the method of finding the complementary function of the homogeneous equation : 15

$$(a_0 D^n + a_1 D^{n-1}D' + \dots + a_n D'^n) z = f(x, y)$$

Solve :

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$$

Or

(*a*) Explain working rule of Lagrange's linear equation is an equation of type : 8

$Pp + Qq = R$, where P, Q and R are functions of x, y, z and :

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$

P.T.O.

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(b) Find the general solution :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + t\frac{\partial z}{\partial t} = xyt$$

2. Solve :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial Q^2} = 0$$

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by the method of separation of variables.

Or

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(a) Find the solution of the wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

such that $y = P_0 \cos pt$, (P_0 is a constant) when $x = l$ and $y = 0$ when $x = 0$.

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(b) Solve :

$$(D + 1)(D + D^1 - 1)z = \sin(x + 2y)$$

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3. Attempt any *two* of the following :

(a) Solve :

$$z = f(x^2 + y^2)$$

(b) Explain the method for solving the equation of the type :

$$f_1(x, p) = f_2(y, q)$$

where :

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$

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(c) Solve :

$$r = a^2 t$$

(d) Use the method of separation of variables to solve the equation :

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$$

given that V = 0 when $t \to \infty$, as well as :

V = 0 at $x = 0$ and $x = l$.