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B—131—2019

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL 2019

MATHEMATICS

Paper VIII

(Ordinary Differential Equations)

(MCQ & Theory)

(Tuesday, 2-4-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for question no. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Question No. **1**.

(*iv*) Negative marking system is applicable for Question No. **1** (MCQs).

(*v*) *All* question are compulsory.

MCQ

1. Choose the *correct* alternative for each of the following : 1 mark each

(*i*) If $p(z)$ is a polynomial of degree 3 with real coefficients, then p has :

(*a*) At least one real root (*b*) At least one imaginary root

(*c*) No real root (*d*) None of these

(*ii*) If r is a root of a polynomial such that $r^3 = 1$ and $r \neq 1$, then :

(*a*) $1 - r + r^2 = 0$ (*b*) $1 + r - r^2 = 0$

(*c*) $1 + r + r^2 = 0$ (*d*) $1 + r^2 = 0$

P.T.O.

(*iii*) The solution ϕ of differential equation $y' + y = e^x$ is given by :

(*a*) $\phi(x) = ce^x$ (*b*) $\phi(x) = ce^{-x}$

(*c*) $\phi(x) = ce^{kx}$ (*d*) None of these

(*iv*) The function ϕ is a solution of $y' = f(x, y)$ where $y \in s$, if :

(*a*) $\phi(x)$ is in s (*b*) $\phi'(s) = f(x, \phi(x))$

(*c*) Both (*a*) and (*b*) together (*d*) None of these

(*v*) The characteristic polynomial of the equation $y'' - y' + 6y = 0$ is :

(*a*) $r^2 + r + 6$ (*b*) $r^2 - r + 6$

(*c*) $r^2 + 6$ (*d*) $r - 6$

(*vi*) If ϕ_1 and ϕ_2 are two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$, then their Wronskian is given by :

(*a*) $W(\phi_1, \phi_2) = \begin{vmatrix} \phi_1 & \phi_2 \\ \phi_2' & \phi_2' \end{vmatrix}$ (*b*) $W(\phi_1, \phi_2) = \phi_1\phi_2' - \phi_1'\phi_2$

(*c*) $W(\phi_1, \phi_2) = \phi_1\phi_2 - \phi_1'\phi_2'$ (*d*) Both (*a*) and (*b*)

(*vii*) A linear differential equation of order n with variable coefficients is an equation of the form $a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \ldots + a_n(x)y = b(x)$, where $a_0, a_1, \ldots, a_n, b$ are complex-valued functions on some real interval I, points where $a_0(x) = 0$ are called :

(*a*) Regular points (*b*) Singular points

(*c*) Both (*a*) and (*b*) (*d*) None of these

(*viii*) If $\phi_1(x) = \sin x$ and $\phi_2(x) = \cos x$, then $W(\phi_1, \phi_2)(x)$, is :

(*a*) 1 (*b*) –1

(*c*) 0 (*d*) 2

(*ix*) If $\phi_1, \phi_2, \ldots, \phi_m$ are any m solutions of the nth order equation $L(y) = 0$ on an interval I, and $c_1, c_2, \ldots, c_m$ are any m constants, then $L(c_1\phi_2 + \ldots + c_m\phi_m) = c_1L_1(\phi_1) + \ldots + c_mL(\phi_m)$ which implies that $c_1\phi_1 + \ldots + c_m\phi_m$ is also a solution, then the trivial solution is the function which is :

(*a*) Identically zero on I (*b*) Identically one on I

(*c*) Identically infinite on I (*d*) None of these (*x*) The solution ϕ of the equation $xy' + y = 0$, such that $y(1) = 1$, is given by :

(*a*) $\phi(x) = x$ (*b*) $\phi(x) = \frac{1}{x}$

(*c*) $\phi(x) = \frac{1}{x^2}$ (*d*) 0

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Let p be a polynomial of degree $n \geq 1$, with leading coefficient 1 (the coefficient of z^n), and let r be a root of p. Then prove that :

$$p(z) = (z - r)\, q(z)$$

where q is a polynomial of degree $n - 1$, with leading coefficient 1.

(*b*) Consider the equation

$$y' + ay = (bx)$$

where a is a constant, and b is a continuous function on an interval I. If x_0 is a point in I and c is any constant, then prove that, the function ϕ defined by

$$\phi(x) = e^{-ax} \int_{x_0}^{x} e^{at}\, b(t)\, dt + ce^{-ax}$$

is a solution of this equation. Every solution has this form.

(*c*) Consider the equation

$$y' + (\cos x)y = e^{-\sin x}$$

(*i*) Find the solution ϕ which satisfies $\phi(\pi) = \pi$.

(*ii*) Show that any solution ϕ has the property that

$$\phi(\pi k) - \phi(0) = \pi k$$

where k is any integer.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Let a_1, a_2 be constants, and consider the equation

$$L(y) = y'' + a_1 y' + a_2 y = 0$$

If r_1, r_2 are distinct roots of the characteristic polynomial p, where

$$p(r) = r^2 + a_1 r + a_2,$$

then prove that, the functions ϕ_1 and ϕ_2 defined by

$$\phi_1(x) = e^{r_1 x}, \ \phi_2(x) = e^{r_2 x}$$

are solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$, and also, if r_1 is a repeated root of p, then the function ϕ_1, ϕ_2 defined by

$$\phi_1(x) = e^{r_1 x}, \ \phi_2(x) = xe^{r_1 x}$$

are solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$.

(*b*) Let ϕ_1, ϕ_2 be two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I, and let x_0 be point in I. Then prove that ϕ_1, ϕ_2 are linearly independent on I if and only if

$$W(\phi_1, \phi_2)(x_0) \neq 0.$$

(*c*) Find all the solutions of non-homogeneous equation

$$y'' - y' - 2y = e^{-x}.$$

4. Attempt any *two* of the following : 5 each

(*a*) Let $\phi_1, \phi_2, \ldots, \phi_n$ be the n solutions of $L(y) = y^{(n)} + a_1(x) y^{(n-1)} + \ldots + a_n(x) y = 0$ on I satisfying

$$\phi_i^{(i-1)}(x_0) = 1, \ \phi_i^{(j-1)}(x_0) = 0, \ j \neq 1$$

Prove that if ϕ is any solution of $L(y) = 0$ on I, there are n constants, $c_1, c_2, \ldots, c_n$ such that

$$\phi = c_1\phi_1 + c_2\phi_2 + \ldots + c_n\phi_n.$$

(*b*) Let x_0 be in I, and let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be any n constants. Prove that there is at most one solution ϕ of

$$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \ldots + a_n(x)y = 0$$

on I satisfying

$$\phi(x_0) = \alpha_1, \ \phi'(x_0) = \alpha_2, \ \ldots, \ \phi^{(n-1)}(x_0) = \alpha_n.$$

(*c*) Consider the equation

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$$

for $x > 0$.

(*i*) Show that there is a solution of the form x^r, where r is a constant.

(*ii*) For two linearly independent solutions for $x > 0$, and prove that they are linearly independent.