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AO—95—2018

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VIII

(Ordinary Differential Equations)

(MCQ & Theory)

(Monday, 2-4-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Q. No. **1** (MCQs).

MCQ

1. Choose the *correct* alternative for each of the following : 1 mark each

(*i*) Let p be a polynomial of degree $n \geq 1$, with leading coefficient 1 (the coefficient of z^n), and let r be a root of p. Then :

(*a*) $p(z) = (z + r)\, q(z)$

(*b*) $p(z) = (z - r)\, q(z)$

(*c*) $p(z) = zq(z - r)$

(*d*) $p(z) = \left(\frac{z - r}{2}\right) q(z)$

P.T.O.

(*ii*) The roots, with multiplicities of the polynomial $z^2 + z - 6$ are :

(*a*) –3, multiplicity 1, 2, multiplicity 1

(*b*) 3, multiplicity 1, 2, multiplicity 1

(*c*) –3, multiplicity 1, –2, multiplicity 1

(*d*) 3, multiplicity 1, –2, multiplicity 1

(*iii*) The solution of the differential equation $y'' + 4y = 0$ is :

(*a*) $\phi(x) = \sin x$ (*b*) $\phi(x) = \cos x$

(*c*) $\phi(x) = \sin 2x$ (*d*) $\phi(x) = \tan 2x$

(*iv*) The solution of the equation $y' + ay = 0$ where a is complex constant and c is any complex number is :

(*a*) $\phi(x) = ce^{ax}$ (*b*) $\phi(x) = ce^{-ax}$

(*c*) $\phi(x) = c/e^{-ax}$ (*d*) None of these

(*v*) Consider the system of equations :

$$iz_1 + z_2 = 1 + i$$

$$2z_1 + (2 - i)z_2 = 1$$

The determinant of the coefficients is :

(*a*) $-1 + 2i$ (*b*) $-1 - 2i$

(*c*) $1 - 2i$ (*d*) All of these

(*vi*) The characteristic polynomial of the equation $y'' + y' - 2y = 0$ is :

(*a*) $r^2 - r - 2$ (*b*) $-r^2 + r - 2$

(*c*) $r^2 + r - 2$ (*d*) $r^2 + r + 2$

(*vii*) The functions $\phi_1(x) = x^2$ and $\phi_2(x) = 5x^2$ are :

(*a*) Linearly dependent

(*b*) Linearly independent

(*c*) Both linearly dependent and linearly independent

(*d*) None of the above

(*viii*) If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0, then :

(*a*) $W(\phi_1, \phi_2)(x) = e^{-a_1(x - x_0)} W(\phi_1, \phi_2)(x_0)$

(*b*) $W(\phi_1, \phi_2)(x) = e^{a_1(x - x_0)} W(\phi_1, \phi_2)(x_0)$

(*c*) $W(\phi_1, \phi_2)(x) = e^{-a_1(x + x_0)} W(\phi_1, \phi_2)(x_0)$

(*d*) $W(\phi_1, \phi_2)(x) = e^{a_1(x + x_0)} W(\phi_1, \phi_2)(x_0)$

(*ix*) If $\phi_1, \phi_2, \ldots\ldots, \phi_n$ are n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \ldots\ldots\ldots + a_n(x)y = 0$ on an interval I, they are linearly independent there if and only if :

(*a*) $W(\phi_1, \ldots\ldots, \phi_n)(x) = 0$ for all x in I

(*b*) $W(\phi_1, \ldots\ldots, \phi_n)(x) \neq 0$ for all x in I

(*c*) $W(\phi_1, \ldots\ldots, \phi_n)(x) \neq 0$ for at least one x in I

(*d*) $W(\phi_1, \ldots\ldots, \phi_n)(x) = 0$ for any x in I

(*x*) A set of functions which has the property that, if ϕ_1, ϕ_2 belong to the set, and c_1, c_2 are any two constants, then $c_1\phi_1 + c_2\phi_2$, belong to the set also, is called a :

(*a*) Linear space of functions

(*b*) Vector space of functions

(*c*) Both (*a*) and (*b*)

(*d*) None of the above

P.T.O.

Theory

2. Attempt any *two* of the following : 5 marks each

(*a*) Consider the equation $y' + ay = b(x)$ where a is a constant, and b is a continuous function on an interval I. If x_0 is a point in I and c is any constant, then prove that the function ϕ defined by :

$$\phi(x) = e^{-ax} \int_{x_0}^{x} e^{at} b(t)\, dt + ce^{-ax}$$

is a solution of this equation.

(*b*) If r is a root of multiplicity m of a polynomial p, deg $p \geq 1$, then prove that $p(r) = p'(r) = \ldots\ldots\ldots = p^{(m-1)}(r) = 0$ and $p^{(m)}(r) \neq 0$.

(*c*) Find all solutions of the equation $y' + 2xy = x$.

3. Attempt any *two* of the following : 5 marks each

(*a*) Let α, β be any two constants and let x_0 be any real number on any interval I containing x_0, then prove that there exists at most one solution ϕ of the initial value problem :

$L(y) = y'' + a_1 y' + a_2 y = 0,\ y(x_0) = \alpha,\ y'(x_0) = \beta.$

(*b*) Let ϕ_1, ϕ_2 be two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I, and let x_0 be any point in I. Then prove that ϕ_1, ϕ_2 are linearly independent on I if and only if :

$$W(\phi_1, \phi_2)(x_0) \neq 0.$$

(*c*) Find all solutions of the following equation :

$$y'' + 4y = \cos x.$$

4. Attempt any *two* of the following : 5 marks each

(*a*) Prove that there exist n linearly independent solutions of :

$$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \dots\dots + a_n(x)y = 0$$

on I.

(*b*) Consider the equation :

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$$

for $x > 0$. Show that there is a solution of the form x^r, where r is a constant.

(*c*) Find two linearly independent solutions of the equation :

$$(3x-1)^2y'' + (9x-3)y' - 9y = 0.$$