This question paper contains **5** printed pages]

W—73—2018

FACULTY OF ARTS/SCIENCE

B.Sc. (First Year) (First Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper-I

(Differential Calculus)

(MCQ+Theory)

(Wednesday, 17-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First **30** minutes for Q. No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR Sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1**.

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) If :

$$y = \frac{\log x}{x}, \text{ then } \frac{d^2y}{dx^2} =$$

(*a*) $\frac{2\log x}{x^3}$ (*b*) $\frac{2\log x - x}{x^3}$

(*c*) $\frac{2\log x - 3}{x^3}$ (*d*) None of these

P.T.O.

(*ii*) If :

$$y = (ax + b)^{-1}, \text{ then } y_n =$$

(*a*) $(-1)^n\, n!\, a^n$

(*b*) $(-1)^n\, n!\, a^{n+1}$

(*c*) $\dfrac{(-1)^n\, n!\, a^n}{(ax+b)^n}$

(*d*) $\dfrac{(-1)^n\, n!\, a^n}{(ax+b)^{n+1}}$

(*iii*) If :

$$y = \cosh x, \text{ then } \frac{dy}{dx} =$$

(*a*) $-\sinh x$

(*b*) $\sinh x$

(*c*) $\cosh x$

(*d*) $-\cosh x$

(*iv*) The length of the tangent to the given curve at any point (x, y) is :

(*a*) $y\sqrt{1+\left(\dfrac{dx}{dy}\right)^2}$

(*b*) $\sqrt{1+\left(\dfrac{dx}{dy}\right)^2}$

(*c*) $y\sqrt{1-\left(\dfrac{dx}{dy}\right)^2}$

(*d*) $y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$

(*v*) Rolle's theorem is applicable if the function is :

(*a*) continuous in $[a, b]$

(*b*) derivable in $]a, b[$

(*c*) such that $f(a) = f(b)$

(*d*) All of these

(*vi*) $\lim\limits_{x \to 1} \dfrac{1 + \log x - x}{1 - 2x + x^2} =$

(*a*) $-\dfrac{1}{2}$

(*b*) $\dfrac{1}{2}$

(*c*) 0

(*d*) 1

(*vii*) lim ($x \log x$) as $x \to 0$ is equal to :

(*a*) -1 (*b*) 1

(*c*) $\frac{1}{2}$ (*d*) 0

(*viii*) The partial derivative of $f(x, y)$ w.r.t. y at (a, b) is given by :

(*a*) $\lim_{k \to 0} \left[\frac{f(a, b+k) - f(a, b)}{k} \right]$

(*b*) $\lim_{k \to 0} \left[\frac{f(a+k, b) - f(a, b)}{k} \right]$

(*c*) $\lim_{k \to 0} \left[\frac{f(a+k, b+k) - f(a, b)}{k} \right]$

(*d*) None of the above

(*ix*) The first order partial derivative of $\log(x^2 + y^2)$ w.r.t. x and y respectively are :

(*a*) $\frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2}$

(*b*) $\frac{x}{(x^2 + y^2)^2}, \frac{y}{(x^2 + y^2)^2}$

(*c*) $\frac{2x}{x^2 + y^2}, \frac{2y}{x^2 + y^2}$

(*d*) $\frac{2x}{(x^2 + y^2)^2}, \frac{2y}{(x^2 + y^2)^2}$

(*x*) Which of the following functions is *not* homogeneous ?

(*a*) $\dfrac{\sqrt{y}+\sqrt{x}}{y+x}$ (*b*) $x^n \sin\left(\dfrac{y}{x}\right)$

(*c*) $\dfrac{\sqrt{x}+\sqrt{y}}{1+xy}$ (*d*) None of these

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If $y = \cos(ax + b)$, then prove that :

$$y_n = a^n \cos\left(ax + b + \frac{n\pi}{2}\right).$$

(*b*) If *u* and *v* are two functions of *x* possessing derivatives of the *n*th order, then prove that :

$$(uv)_n = u_n v + nc_1\, u_{n-1}\, v_1 + nc_2\, u_{n-2}\, v_2 + \ldots\ldots + nc_n\, uv_n$$

(*c*) Find the equations of tangent and normal at $\theta = \dfrac{\pi}{2}$ to the curve :

$$x = a(\theta + \sin\theta),\ \ y = a(1 + \cos\theta).$$

3. Attempt any *two* of the following : 5 each

(*a*) If two functions $f(x)$ and $F(x)$ are derivable in $[a, b]$ and $F'(x) \neq 0$ for any value of x in $[a, b]$, then prove that there exists at least one value '*c*' of *x* belonging to $]a, b[$ such that :

$$\frac{f(b) - f(a)}{F(b) - F(a)} = \frac{f'(c)}{F'(c)}.$$

(*b*) If $f(x) = (x-1)(x-2)(x-3)$; $x \in [0, 4]$, then find the value of c.

(*c*) Find :

$$\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x}.$$

4. Attempt any *two* of the following : 5 each

(*a*) If $z = f(x, y)$ is a homogeneous function of x, y of degree n, then prove that :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

(*b*) If :

$$u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}; \quad xy \neq 0$$

then prove that :

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}.$$

(*c*) Verify the Euler's theorem for :

$$z = ax^2 + 2hxy + by^2.$$