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W—74—2018

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Third Year) (Sixth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

MATHEMATICS

Paper-XVIII

(Topology)

(Wednesday, 17-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define basis for a topology.

(*b*) Define well-ordered set.

(*c*) Define the product topology.

(*d*) Define coordinate function.

(*e*) Define closure and interior of a set.

(*f*) Define connected set.

2. Attempt any *two* of the following : 5 each

(*a*) Let X be a topological space. Suppose **C** is a collection of open sets of X such that for each x in X and each open set U of X, there is an element $\subset$ of **C** such that $x \in C \subset U$. Then prove that **C** is a basis for the topology of X.

(*b*) Show that the set $Z_+ \times Z_+$ is countably infinite.

(*c*) Let X be a set, let τ_f be the collection of all subsets U of X such that $X - U$ either is finite or is all of X. Then show that τ_f is a topology on X.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) If A is a subspace of X and B is a subsapce of Y, then prove that the product topology on A × B is the same as the topology A × B inherits as a subsapce of X × Y.

(*b*) Define the subspace topology. If B is a basis for the topology of X, then prove that the collection :

$$\mathbf{B}_Y = \{B \cap Y \mid B \in \mathbf{B}\}$$

is a basis for the subspace topology on Y.

(*c*) Define open map. Show that maps $\pi_1 : X \times Y \to X$ and $\pi_2 : X \times Y \to Y$ are open maps.

4. Attempt any *two* of the following : 5 each

(*a*) Let Y be a subspace of X, let A be a subset of Y, let $\bar{A}$ denote the closure of A in X. Then the closure of A in Y equals $\bar{A} \cap Y$.

(*b*) Let X and Y be topological spaces, let $f : X \to Y$. Then show that the following are equivalent :

(*i*) f is continuous

(*ii*) For each subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.

(*iii*) For each closed set B in Y, the set $f^{-1}(B)$ is closed in X.

(*c*) Show that the product of two Hausdorff spaces is Hausdorff.