This question paper contains **3** printed pages]

V—81—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2017

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VII

(Group Theory)

(Tuesday, 14-11-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Question No. **1** (MCQ) and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Question No. **1**.

(*iv*) Negative marking system is applicable for Question No. **1** (MCQ).

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) Let S and T be any sets; define $\tau : S \times T \to S$ by $(a, b)\tau = a$ for any $(a, b) \in S \times T$, then τ is called :

(*a*) Projection of $S \times T$ on S

(*b*) Projection of $S \times T$ on T

(*c*) Image of $S \times T$ on S

(*d*) Pre-image of $S \times T$ on S

(*ii*) If a and b are relatively prime, we can find integers m and n such that :

(*a*) $ma + nb = 2$ (*b*) $ma + nb \neq 2$

(*c*) $ma + nb \neq 1$ (*d*) $ma + nb = 1$

P.T.O.

(*iii*) Let S = $\{x_1, x_2, x_3\}$ and let T = S. Let $\sigma : S \to S$ be define by $x_1\sigma = x_2$, $x_2\sigma = x_3$, $x_3\sigma = x_1$; and $\tau : S \to S$ by $x_1\tau = x_1$, $x_2\tau = x_3$, $x_3\tau = x_2$ then $x_1(\sigma . \tau)$ = ...

(*a*) x_1 (*b*) x_2

(*c*) x_3 (*d*) None of these

(*iv*) If a, b in the group G, then the equations $a. x = b$ and $y . a = b$ have unique solutions for ....

(*a*) x in G (*b*) y in G

(*c*) Both (*a*) and (*b*) (*d*) None of these

(*v*) If H is a subgroup of G, then the number of distinct right cosets of H in G is called ....

(*a*) Order of H in G (*b*) Index of H in G

(*c*) Normal of H in G (*d*) Congruence of H in G

(*vi*) If n is a positive integer and a is a relatively prime to n, then :

(*a*) $a^{\phi(n)} \equiv 1 \bmod n$ (*b*) $a^p \equiv 1 \bmod p$

(*c*) $a^p \not\equiv 1 \bmod p$ (*d*) None of these

(*vii*) Let G be the group S_3 and let H be the subgroup $\{e, \phi\}$, then the number of right cosets of H in G are ....

(*a*) 2 (*b*) 3

(*c*) 0 (*d*) 4

(*viii*) A homomorphism ϕ of G into $\overline{G}$ with kernel K_ϕ is an isomorphism of G into $\overline{G}$ if and only if :

(*a*) $K_\phi = 0$ (*b*) $K_\phi = \phi$

(*c*) $K_\phi = (e)$ (*d*) $K_\phi = \{1, 2\}$

(*ix*) If the groups G, G^*, G^{**} are isomorphic, then :

(*a*) $G \approx G$

(*b*) $G \approx G^*$ implies $G^* \approx G$

(*c*) $G \approx G^*$, $G^* \approx G^{**}$ implies $G \approx G^{**}$

(*d*) All of the above

(*x*) The product of two even permutations is ....

(*a*) an even permutation (*b*) an odd permutation

(*c*) not a permutation (*d*) None of these

Theory

2. Attempt any *two* of the following : 5 each

   (*a*) If $a \equiv b$ mon n and $c \equiv d$ mod n, then prove that $a + c \equiv b + d$ mon n and $ac \equiv bd$ mod n.

   (*b*) If G is a group, then prove that :

   - (*i*) The identity element of G is unique.
   - (*ii*) For all $a, b \in$ G, $(a.b)^{-1} = b^{-1}. a^{-1}$.

   (*c*) If G is a group such that $(a.b)^2 = a^2. b^2$ for all $a, b \in$ G show that G must be abelian.

3. Attempt any *two* of the following : 5 each

   (*a*) Prove that HK is a subgroup of G if and only if HK = KH.

   (*b*) If H is a non-empty finite subset of a group G and H is a closed under multiplication, then prove that H is a subgroup of G.

   (*c*) If N is a normal subgroup of G and H is any subgroup of G, prove that NH is a subgroup of G.

4. Attempt any *two* of the following : 5 each

   (*a*) If ϕ is a homomorphism of G into $\bar{G}$ with kernel K, then prove that K is a normal subgroup of G.

   (*b*) Prove that every group is isomorphic to a subgroup of A(S) for some appropriate S.

   (*c*) If θ is the permutation represented by $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$ and ψ is the permutation represented by $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$, then find $\theta\psi$ and $\psi\theta$.