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B—108—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VII

(Group Theory)

(MCQ+Theory)

(Saturday, 30-3-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Question No. **1** (MCQ) and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Question No. **1** (MCQs).

(MCQ)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) The mapping τ of S into T is said to be a one-to-one mapping if :

(A) whenever $s_1 \neq s_2$, then $s_1 \tau \neq s_2 \tau$

(B) whenever $s_1 = s_2$, then $s_1 \tau = s_2 \tau$

(C) whenever $s_1 \neq s_2$, then $s_1 \tau = s_2 \tau$

(D) whenever $s_1 = s_2$, then $s_1 \tau \neq s_2 \tau$

(*ii*) When are the two integers a and b said to be relatively prime ?

(A) $(a, b) = a$ (B) $(a, b) = b$

(C) $(a, b) = 0$ (D) $(a, b) = 1$

P.T.O.

(*iii*) If G is a group, $a \in$ G, then for any itnegers m and n, $a^m.a^n$ =

(A) a^{mn} (B) a^{m+n}

(C) a^{m-n} (D) $a^{m/n}$

(*iv*) If G is a group and a, $b \in$ G, then $(a.b)^{-1}$ = ?

(A) $a^{-1}.b^{-1}$ (B) $b^{-1}.a^{-1}$

(C) $a.b$ (D) $b.a$

(*v*) If G is a group and H is a subgroup of G, then for any a, $b \in$ G when does the relation $a \equiv b$ mod H hold :

(A) $a\ b \in$ H (B) $a^{-1}\ b^{-1}, \in$ H

(C) $ab^{-1} \in$ H (D) All of these

(*vi*) If H is a subgroup of a finite group G, then what is index of H in G ?

(A) The number of distinct right cosets of H in G

(B) The order of H

(C) The order of H/G

(D) None of the above

(*vii*) If G is a finite group and N is normal subgroup of G, then O(G/N) = ?

(A) O(G) + O(N) (B) O(G) – O(N)

(C) O(G) × O(N) (D) O(G)/O(N)

(*viii*) If ϕ is an isomorphism from G into $\bar{G}$, then which of the following is *true* ?

(A) ϕ is a homomorphism (B) ϕ is one-to-one

(C) Both (A) and (B) (D) None of these

(*ix*) What is an automorphism on a group G ?

(A) A homomorphism of G into itself

(B) A homomorphism of G onto itself

(C) An isomorphism of G into itself

(D) An isomorphism of G onto itself

(*x*) The product of two odd permutation is :

(A) An even permutation (B) An odd permutation

(C) Not a permutation (D) None of these

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If $\sigma : S \to T$, $\tau : T \to U$ and $\mu : U \to V$, then prove that :

$$(\sigma \, o \, \tau) \, o \, \mu = \sigma_o(\tau \, o \, \mu).$$

(*b*) For $S = \{x_1, x_2, x_3\}$ and $\phi, \psi \in S_3$ given by :

$\phi : x_1 \to x_2$

$x_2 \to x_1$

$x_3 \to x_3$

and the mapping

$\psi : x_1 \to x_2$

$x_2 \to x_3$

$x_3 \to x_1$

find $\phi.\psi$ and $\psi.\phi$.

(*c*) Prove that if G is an abelian group, then for all $a, b \in G$ and all integers n,

$$(a.b)^n = a^n.b^n$$

3. Attempt any *two* of the following : 5 each

(*a*) Prove that a non-empty subset H of the group G is a subgroup of G if and only if :

(*i*) $a, b \in H$ implies that $a\,b \in H$

(*ii*) $a \in H$ implies that $a^{-1} \in H$.

(*b*) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

(*c*) Show that the intersection of two normal subgroups of G is a normal subgroup of G.

P.T.O.

4. Attempt any *two* of the following : 5 each

(*a*) If ϕ is a homomorphism of a group G into a group $\overline{G}$, then prove each of the following :

(*i*) $\phi(e) = \overline{e}$, the unit element of $\overline{G}$.

(*ii*) $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$.

(*b*) Prove that every group is isomorphic to a subgroup of A(S) for some appropriate S.

(*c*) Find the orbits and cycles of the permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}.$$