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PA—54—2024

FACULTY OF SCIENCE/ARTS

B.A./B.Sc. (First Semester) EXAMINATION

APRIL/MAY, 2024

(New Pattern)

MATHEMATICS

Paper-I

(Calculus-I : Differential Calculus)

(Friday, 19-04-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 Hours *Maximum Marks—40*

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. State and prove that Leibnitz's theorem for nth derivative of the product of two functions. 15

Or

(*a*) Let $f(x + h)$ be a function of h (x being independent of h) which can be expanded in powers of h and the differentiable any number of times, then prove that : 8

$$f(x + h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \ldots\ldots\ldots\ldots$$

$$+ \ldots\ldots\ldots\ldots + \frac{h^n}{n!} f^n(x) + \ldots\ldots$$

P.T.O.

(b) If the relation between subnormal SN and subtangent ST at any point S on the curve : 7

$$by^2 = (x + a)^3 \text{ is}$$

$$\lambda(SN) = \mu\,(ST)^2, \text{ then find the value of } \frac{\lambda}{\mu}$$

2. State and prove that Euler's theorem on homogenous function. Also show that : 15

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{4}\sin 2u = 0$$

$$\text{if } U = \cot^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$$

Or

(a) State and prove Rolle's theorem. 8

(b) If $f(x) = (x-1)(x-2)(x-3)$, $x \in [0, 4]$, then find C by using Lagrange's mean value theorem. 7

5 each

3. Answer the following (any *two*) :

(*a*) If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then find $\frac{d^2y}{dx^2}$.

(*b*) Expand $\cos x$ by Maclaurin's series.

(*c*) Using Lagrange's mean value theorem, show that :

$$\frac{x}{1+x} < \log(1+x) < x, \; x > 0.$$

(*d*) If $z = \log(x^2 + y^2)$, then show that :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$